Non-biased estimation of modal parameters of multi-component ambient vibrations in high-rise buildings using a self-governed random decrement technique

Fatima Nasser[1], Zhong-yang Li[1], Nadine Martin[1] and Philippe Gueguen[2]
[1]Gipsa-lab, Departement Images Signal
BP 46 – 961 F-38402 Saint Martin d'Heres, France
firstname.lastname@gipsa-lab.grenoble-inp.fr

[2]ISTerre, Univ. Grenoble /CNRS/IFSTTAR
BP 53, 38041 Grenoble cedex 9, France
Pilippe.gueguen@obs.ujf-grenoble.fr

Abstract

The estimation of modal parameters of a structure by applying the random decrement technique has been introduced since late 1960's, and is widely used because of its simplicity and rapidity. Since this technique can be applied to mono-component signals only, a filtering process is a necessary preliminary step for getting the random decrement signature when applying on multi-component signals. But filtering is the most critical step with a high influence on the accuracy of the modal parameter estimation, especially when the signal is composed of very closely-spaced modes. In this paper, we propose to apply the random decrement technique directly on a multi-component signal without the filtering step, in order to provide what we called a multi-mode signature. The idea is then to propose an adapted method to separate the different modes in this multi-mode signature. For that purpose the modal parameters are first roughly approximated in a non-optimal way and with a bias from a time-frequency distribution. Based on this initialization, a final non-biased estimation is achieved via maximum likelihood estimator. The results conducted in this paper, over simulated and real world ambient vibrations on high rise buildings, suggest positively that the proposed method is totally autonomous and is capable of yielding very good estimation results.

1. Introduction

In civil engineering, the study of the dynamic behaviour of the building plays a significant role in detecting its possible damages, and determining its safety and reliability.
The dynamic properties of the building are directly related to the mass and the stiffness, and the modal parameters are the basic contributor in giving information about the inherent dynamic properties of the building. So that, knowing the modal parameters would provide sufficient information about these two physical properties. As such, it is of high importance to estimate these parameters with high precision.

However, the signals measured on the buildings under study are the response of these buildings to the ground excitation which is assumed to be white Gaussian noise. To estimate the modal parameters for such type of signals, the impulse response should be firstly estimated, and thus a deconvolution method is a must. Here comes the motivation of using the Random Decrement Technique (RDT).

The RDT was developed by Henry Cole [(1)] in the late 1960's. Driven by the motive of online failure detection by monitoring the damping of a structure excited by white noise. Cole intuitively came up with a process of averaging the time segments with the same initial conditions to get a function proportional to the free decay.

When a large enough number of selected samples from a very long time history conditions are averaged, the random component of response averages out to zero and what survives is only the free decay, or the impulse response, or what is known as the Random Decrement Signature (RDS) [(1)].

This technique was given a mathematical basis later on by Vandiver et al in 1982 [(2)] , who showed that for a practical case of a linear time-invariant system excited by a zero-mean, stationary, Gaussian random process, the RDS of the output is shown to be proportional to the autocorrelation of the process [(2)].

The modal parameters are estimated using the RDS [(3, 4)]. Thus in order to obtain credible modal parameter estimations, one would have to pay great attention on the estimation of the RDSs themselves.

In practice, the RDT is applicable over mono-component signals only, thus a filtering process is a necessary preliminary step for getting the RDS when applying over multi-component signals.

During the study of the RDT, it is observed that filtering is one of the most delicate and critical processes, and it is difficult to be performed firmly, especially when the multi-component signal is composed of very closely spaced frequency modes.

Firstly, we intended to tune the filtering bandwidth manually, as it is prevalent in most cases, e.g. [(5)], however, the manual tuning of the filtering process is found to be very time consuming with high influence on the modal parameter precision.

Then in Surveillance 7 [(6)], we proposed to set the filter bandwidth automatically. This proposition has made the estimation of the modal parameters faster, easier, reliable, and independent of user interaction. But it doesn't however rule out the user know how completely, because there are still several factors which require user intervention, for e.g. the choise of the filter type and the filter order. From here, we thought that it would be great if this step could be totally skipped.

Even though, the RDT has been widely investigated since its development [(7, 8)], however, it had been rarely carried out without filtering. This work responds to this shortage and proposes a novel approach to extend the applicability of the RDT for modal parameter estimation without the necessity of the filtering process. This makes the originality of this paper.

The application of the RDT over multi-component signals without passing by the filtering process leads to the estimation of what we called multi-mode RDS.

Working with the multi-mode RDS allowed the proposition of a new method for the modal parameter estimation, mainly the frequency and the damping ratio.

The proposition states first to roughly approximate the modal parameters, using a non-optimal way. Based on this initialization step, a final non-biased estimation is achieved from a maximum likelihood estimator.

Nevertheless, when dealing with the RDT, one should not turn a blind eye toward a very important aspect, which is the processing parameters of this technique, that play a significant role in the accuracy of the RDS extraction and thus the modal parameter estimation. Proceeding from this fact, an intense study of the influential parameters of the RDT, mainly on the damping ratio estimation, has been carried out through this work, and will be briefly presented throughout this paper.
The rest of this paper is organized as follows: the problematic of this paper is formulated in section 2. Section 3 shows a brief introduction to the RDT and the estimation of the multi-mode RDS. The proposed method is then explored in section 4. Section 5 presents briefly the processing parameters of the RDT and their effect on the modal parameter estimation. Section 6 illustrates the obtained results over simulated and real world signals. Results are finally discussed in section 7 in terms of relevance and perspective.

2. Problem formulation

The signal under study in this paper is considered as

$$Y[n] = h\ [n] * P[n] + e[n], \tag{1}$$

The index n is interpreted as discrete time, $h\ [n] = \sum_{k=1}^{N} h_k[n]\ \ \forall k \in [1, N]$ where N is the number of modes in the signal and being equal to 1 for single degree of freedom systems (SDOF), $Y[n]$ corresponds to the ambient vibration of the building, $*$ indicates the convolution between the impulse response $h\ [n]$ of that building and the seismic noise $P(t)$ that is assumed to be white Gaussian noise of zero mean and unknown variance. The impulse response is characterized by its natural frequency (f_k), and damping ratio (ξ_k), and it is defined as

$$h_k[n] = \frac{-1}{2\pi f_k}\left(e^{-\xi_k 2\pi f_k n}\right)\sin(\omega_{Dk} n), \tag{2}$$

where $\omega_{Dk} = 2\pi f_k \sqrt{1-\xi_k^2}$ the pseudo pulsation. $e(t)$ is an additive white Gaussian noise of zero mean and unknown variance.
In order to estimate the modal parameters of such types of signals we have to deal with the impulse response separately, here comes the motivation behind using the RDT, since it is a method able to estimate a signature similar to the impulse response of the building known as the RDS. The way of estimating the RDS by using the RDT is explained in section 3.

3. Multi-mode Random Decrement Signature

The RDT is a method which transforms the stochastic process $Y[n]$, that is assumed to be stationary process, into RDS.
The RDS is defined as the mean value of a stochastic process on a condition of the process itself [3, 4],

$$D_Y[\tau] = E\left[Y[n+\tau]\ \middle|T_{Y[n]}\right], \tag{3}$$

An RDS is referred to $D_Y(\tau)$, where τ indicates the process time lag, $E\left[Y[n+\tau]\ \middle|T_{Y[n]}\right]$ is the mean value of $Y[n+\tau]$ being the process from which the mean value is calculated, on $T_{Y[n]}$ being the process where the condition is fulfilled.
For a time series, the RDS can be estimated unbiased as an empirical mean

$$\hat{D}_Y[\tau] = \frac{1}{M}\sum_{i=1}^{M}(y[n_i+\tau]\ |T_{y[n_i]}), \tag{4}$$

where $\hat{D}_Y(\tau)$ is the estimated RDS, $y[n]$ is the measurement and M is the number of points in the process which fulfils the triggering condition $\forall i \in [1, M]$.
The initial condition of the time segments in the averaging process are presented by the triggering condition at the time lag $\tau = 0$. Asmussen [3] has defined the different triggering conditions used as a specific formulation of the applied general triggering condition,

$$T_{Y[n]} = \left\{a_1 \le Y[n] < a_2, b_1 \le \dot{Y}[n] < b_2\right\}. \tag{5}$$

where a_1, a_2, b_1 and b_2 are the triggering levels, and $\dot{Y}[n]$ is the discrete time differentiation of $Y[n]$.
Asmussen [3], as well, has presented the most frequently used triggering conditions as level crossing, $T^L_{Y(t)} = \{Y(t) = a\}$, positive point, $T^P_{Y(t)} = \{a_1 \le Y(t) > a_2\}$, local extrema, $T^E_{Y(t)} = \left\{a_1 \le Y(t) > a_2, \dot{Y}(t) = 0\right\}$, and zero-up crossing, $T^Z_{Y(t)} = \left\{Y(t) = a_2, \dot{Y}(t) > 0\right\}$. The number of triggering points controls the estimation time and the accuracy of the estimates.
All the modes of the input signal are preserved in the estimated RDS, so that if the input is single-mode signal, then the estimated RDS will contain only a single mode, and vice versa. Since we proposed to apply the RDT directly over the multi-component ambient vibration signal under study, so the estimated RDS will be the multi-mode RDS that contains the same natural frequencies (f_k), and damping ratios (ξ_k) as the impulse response of the signal under study presented in equation (2).
We assume that the model of the multi-mode RDS is defined as

$$y[m] = s[m] + e_{RDS}[m], \tag{6}$$

with $y[m]$ be a discrete time process consisting of a deterministic multi-component process $s\,[m]$, embedded in the residual of the RDT algorithm which can be considered as an additive white Gaussian noise $e_{RDS}[m]$ with zero mean and unknown variance,

$$s\,[m] = \sum_{k=1}^{N} s_k[m] \text{ with } s_k[m] = A_{0k}e^{-2\pi f_k \xi_k m}\cos(2\pi f_k m\sqrt{1-\xi_k^2}), \tag{7}$$

where m is interpreted as the time index of the RDS, with $1 \le k \le N$ and A_{0k} being the initial amplitude of each mode.

4. A new modal parameter estimation algorithm

Despite of their rapidity and simplicity, Fourier transform based approaches are hardly able to estimate and track the model parameters with no bias. Parameteric or model-based estimators can give significant improvements in the time-frequency resolution at the expense of a higher computational complexity.
Accordingly, a non-biased estimation is achieved by using the parametric Maximum Likelihood Estimator (MLE), this is a deterministic approach that is maximized by means of a stochastic technique based on the simulated annealing algorithm. However, good starting values of the parameters are helpful in reducing wanders among local minima. Therefore, we initialize the parameter vector of MLE by using the non-parametric spectral based approach.

4.1 Maximum Likelihood Estimation

This section describes a parametric spectral estimator method using MLE in order to find the optimum set of parameters that minimize the error function. The aim is to reduce the bias in the frequency resolution of the conventional methods by using the characteristics associated to the MLE. This will allow the computational of higher accurate damping ratios, and increasing the frequency resolution.
The parameters of each mode of (7) form a vector,

$$\boldsymbol{\varphi}_k = \left[\boldsymbol{\Phi}_1^T, \ldots, \boldsymbol{\Phi}_k^T \right] = \left[A_{0k}, \xi_k, \phi_{0k}, f_k \right]^T , \tag{8}$$

where T indicates the transpose, and ϕ_{0k} the initial phase of each mode.
We consider the MLE of $\boldsymbol{\varphi}$ being equivalent to the minimization of a least square approach under the assumption of white Gaussian additive noise. Thus it results in the following non-linear equation

$$\hat{\boldsymbol{\varphi}} = \arg\max_{\boldsymbol{\varphi} \in \Re^{4 \times k}} l_{MLE}(\boldsymbol{\varphi}) = \arg\min_{\boldsymbol{\varphi} \in \Re^{4 \times k}} l_{LS}(\boldsymbol{\varphi}), \tag{9}$$

$$l_{LS}(\boldsymbol{\varphi}) = \arg\min_{\boldsymbol{\varphi} \in \Re^{4 \times k}} \sum_{m=-L/2}^{L/2} \left| y[m] - s[m] \right|^2 , \tag{10}$$

where L is the sample number.
Direct minimization of (10) is difficult due to the high non-linearity of the function and the parameter number. Classical optimization techniques such as gradient descent, Gauss-Newton and EM algorithm do not ensure convergence to the global minimum when local minima are numerous.
This problem can be overcome with meta-heuristic approaches, and in particular, with the simulated annealing algorithm [9]. The simulated annealing technique is efficient when a desired global extremum is hidden in many local extrema. The simulated annealing has an analogy with thermodynamics where metal cools and anneals. In the

same way, after an initialization of the parameter vector to estimate as will be explained in section 4.2, an iterative loop, controlled by a scalar referred to as a temperature, generates a new candidate of the vector that minimizes the cost function, namely the least square function $l_{LS}(\boldsymbol{\varphi}$. This candidate is accepted or not according to the Metropolis acceptance rule.

4.2 Spectral based estimation

In the aim of obtaining a rough estimation of the modal parameters, the classical, non-parametric spectral based approach from time-frequency distribution will be used. This step will serve as an initialization step to provide a priliminary estimation for the MLE approach.

Accordingly, the usage of two windows centered at $\frac{1}{3}L_{rds}$ with 50% overlapping is enough. Those windows are each of length $\frac{2}{3}L_{rds}$, with L_{rds} being the length of the estimated multi-mode RDS as shown in fig. 1. Thus two periodograms at two time instants $t_1 = \frac{1}{3}L_{rds}$ and $t_2 = \frac{2}{3}L_{rds}$ are calculated respectively by applying the Short Time Fourier Transfom (STFT) of two windows $\left[0, \frac{2}{3}L_{rds}\right]$ and $\left[\frac{1}{3}L_{rds}, L_{rds}\right]$. The average of these two periodograms yields the Welch spectral estimator of the multi-mode RDS, from which for each detected frequency peak a related damping ratio can be calculated using the magnitude of the detected peak at the two different instants t_1 and t_2 .

$$\alpha(f_k) = \frac{\log_e\left(\frac{A_1(f_k)}{A_2(f_k)}\right)}{\frac{L_{rds}}{3}} \tag{11}$$

with $\alpha(f_k) = 2\pi f_k \xi(f_k)$, and A_1, A_2 are the amplitudes of the detected f_k at instants t_1 and t_2 respectively.

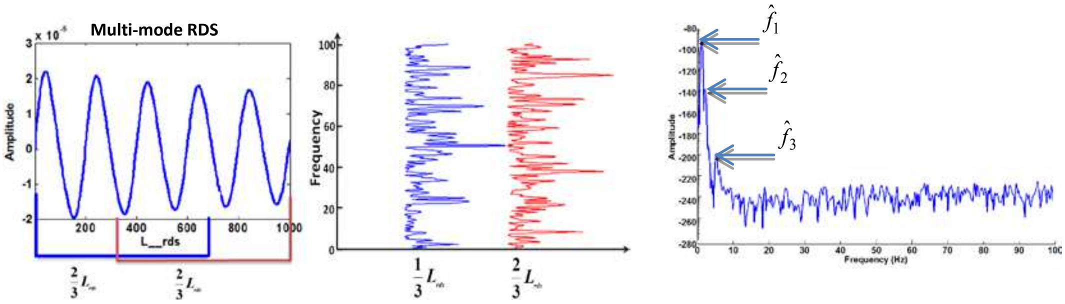

Figure 1. The estimated multi-mode RDS (left), the periodograms calculated using the STFT with a window length equal to $\frac{2}{3}L_{rds}$ at two time instants $t_1 = t_2 = \frac{1}{3}L_{rds}$ (middle), and the average of the two frames that results in Welch spectral estimation (right)

One of the drawbacks of the STFT approach is that the frequency resolution is limited due to the windowing of the RDS.
The usage of the RDT with the multi-mode RDS has posed some problems which necessitates the intense study of the processing parameters of this technique, that play a significant role in the accuracy of the RDS extraction and thus the modal parameter estimation.

5. RDT Processing parameter study

For the efficient use of the RDT, the data processing parameters associated with the technique need to be understood. Some of these parameters play a significant role in accurate estimation of modal parameters; the parameter settings in this work are judged depending on their influence on the accuracy of the estimated damping ratio. Based on this judgment, the optimum parameter settings will be chosen for implementation of RDT over multi degree of freedom systems.
On one hand, the length of the RDS, $L_{rds}\ (points)$, and the length of the signal under study, $L_{sig}\ (points)$, are the operational parameters of RDT that are manipulated according to the user requirements. Those parameters should be carefully chosen in order to study their effect on the damping ratio estimation.
On the other hand, the sampling frequency of the signal under study, F_s, is a characterizing parameter of the input, and it is taken into account in the behaviour of the algorithm, and the accuracy of the parameters to estimate. That are in turn, the natural frequency, f_k, and the damping ratio, ξ_k, of each mode k in the signal under study.
Then the influence of 2 parameters are to be studied according to the estimation of 2 modal parameters, and taking into consideration the signal sampling frequency.
When the natural frequency is normalized by the signal sampling frequency, both L_{rds} and L_{sig} can then be normalized in terms of their number of periods, such as, $N_{rds} = \frac{L_{rds}}{R(f_k)}\ (periods)$, and $N_{sig} = \frac{L_{sig}}{R(f_k)}\ (periods)$, with $R(f_k) = \frac{F_s}{f_k}\ (points/period)$.
Subsequently, the parameters to be studied are N_{rds} and N_{sig}. Accordingly, those processing parameters and their impact on the accuracy of damping ratio estimate using RDT were evaluated firstly on SDOF systems.
The conclusion of the processing parameter study is based on 20 independent noise realizations. N_{sig} was chosen to range from $500\ \ periods$ to $4000\ periods$. For $N_{rds}(f_k)$ we chose it to range from 3 to 12 cycles; along with a damping ratio ranges between 1% to 5% where values of real world signals can be found. The SDOF systems analyzed experimentally also have estimated different signal to noise ratio (SNR) for the additive noise; $SNR_{add} = 0{,}5{,}15 dB$.

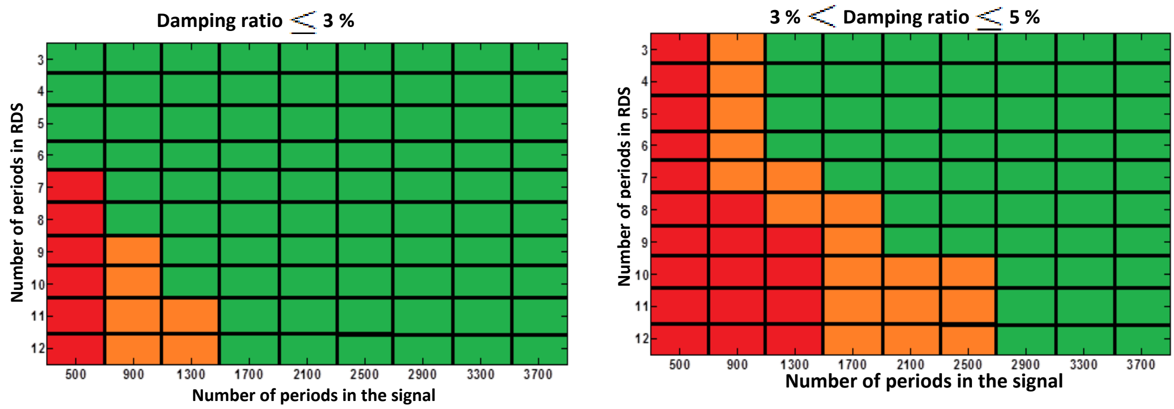

Figure 2. Good regions (green), intermediate regions (orange) and bad regions (red) when setting the number of periods of the RDS according to the signal

In the application of the RDT, N_{sig} and $N_{rds}(f_k)$ should be chosen as the result of a compromise. On one hand, the signal length should be neither too long in order not to loose the time resolution of the signal, nor too short in order to assure that it contains a large number of averaged segments that yield the exponentially decaying form. On the other hand, the length of the signature should be large enough to assure that it is of exponentially decaying form, however, not too long in order to prevent the beating phenomena that appears with very long RDS. Noting that it should not be shorter than two periods, otherwise the decaying phenomena will be deteriorated.
From the results of this study that are summarized in fig. 2, it is always recommended to be in the green region which yields an acceptable estimation error of the modal parameters of $Estim_{error} \leq 30\%$. It is not recommended to be in the orange region that gives a high estimation error of $30\% < Estim_{error} \leq 50\%$, however, if it is unavoidable to be in that region it should be expected that the estimations might be less reliable, and for sure it is totally unrecommeded to be in the red region since it associates a very high, unacceptable estimation error of $Estim_{error} > 50\%$.

6. Application on ambient vibration signals

Buildings are permanently excited by some natural solicitation sources, for example, the background seismic noise from the earth, the natural phenomena around the building, internal source like human steps, vihicles, and the different types of rotating machineries in the buildings, and the natural activity of the earth. These different excitations induce various types of vibrations in the building, which are referred to as ambient vibrations.
This section is aimed at the discussion of the accuracy and efficiency of the proposed algorithm in this paper for the modal parameter estimation over simulated ambient vibration signals, and real-world ambient vibration signals recorded on real-world buildings. This discussion is guided by the experience gained from the intense study over the RDT processing parameters, explained in section 5, using the “optimum” parameter settings needed to yield reliable estimations.

6.1 Analysis of simulated ambient vibration signals

To study the relevancy of the proposed algorithm on modal parameter estimation, three simulated signals have been devised. These signals are of different configurations, as shown in Table 1, to assure the general functionality of the proposition.
Based on the intense study of the processing parameters of the RDT briefed in section 5, N_{sig} and N_{rds} for all the three simulated signals were selected from the recommended green zone in fig. 1, as 5 and 3100 periods respectively.
It is worth noting as well that the triggering condition used in this section was the positive point type, this was encouraged by the experimental study we've done over the RDT processing parameters, and by what was recommended by Assmusen [10] to analyse ambient data. The SNR_{add} was chosen to be $15dB$.

Table 1. Three simulated signals Sig_1, Sig_2 and Sig_3 with different configurations

	Sig_1	Sig_2	Sig_3
$f_k(Hz)$	$[2,5]$	$[1,3,5,7]$	$[2.1,2.2]$
$\xi_k(\%)$	$[3,4]$	$[1,1.5,2,2.5]$	$[1,2]$
$F_s(Hz)$	200	200	200
$Signal\_Length(Points)$	310,000	180,000	180,000
N_{modes}	2	4	2

Table 2. The estimated modal parameters of Sig_1, Sig_2 and Sig_3 along with their normalized error reported in parenthesis

	Sig_1		Sig_2		Sig_3	
	$\hat{f}_k$ **(Hz)**	$\hat{\xi}_k$**(%)**	$\hat{f}_k$ **(Hz)**	$\hat{\xi}_k$**(%)**	$\hat{f}_k$ **(Hz)**	$\hat{\xi}_k$**(%)**
1st mode	1.99 **(0.005)**	3.05 **(0.01)**	0.99 **(0.01)**	1.09 **(0.09)**	2.09 **(0.004)**	0.61 **(0.39)**
2nd mode	4.87 **(0.02)**	3.64 **(0.09)**	3.00 **(0.00)**	1.88 **(0.25)**	2.20 **(0.00)**	2.44 **(0.22)**
3rd mode	N/A	N/A	4.87 **(0.02)**	2.50 **(0.25)**	N/A	N/A
4th mode	N/A	N/A	6.36 **(0.09)**	2.70 **(0.08)**	N/A	N/A

Table 2 summarizes the results of the modal parameter estimation for the three simulated signals. Parameter estimations shown in the table are the natural frequencies and the damping ratios with normalized errors reported in parantheses.
As shown in the table, both natural frequencies and damping ratios were estimated with better precision for higher energy modes (modes in lower frequencies), except for Sig_3

at which the modes are very closely spaced in frequency. This result is expected when dealing with optimization methods like the simulated annealing, as it is very difficult to estimate the correct parameters of the weak energy modes with such types of methods.

$$Energy(f_k) \propto \frac{1}{f_k^{\,2} \times \xi_k}, \tag{12}$$

As presented by equation (12), it is not surprising that for all the signals of this section, the estimations of the last mode is not as good as the other modes in lower frequencies, as its energy is much lower than the others which makes it indifferentiable from the noise.

Nevertheless, all the measures of normalized errors indicate that on average the proposed model exhibit very good estimation performance.

6.2 Analysis of real-world ambient vibration signals

The signal of this part is a real-world ambient vibration signal, recorded at the top of the 13th floor city hall building of Grenoble (France), fig. 3, using multiple sensors placed on several stories to measure simultaneously the vibrations in three directions, longitudinal, transverse and vertical. This signal is sampled at 200 Hz.

For this signal we study the analysis uniquely over the data in the longitudinal direction. Based on the information provided to us by the geophysicists, we will be focusing on the first three modes represented respectively as, the first longitudinal at 1.15 Hz, the first transverse at 1.2 Hz, and the first torsion mode at 1.44 Hz.

Figure 3. Grenoble city hall, France.

Table 3. The estimated modal parameters of the signal recorded at the top of Grenoble city hall building

	$\hat{f}_k$ **(Hz)**	$\hat{\xi}_k$**(%)**
1st mode	1.15	2.00
2nd mode	1.20	2.00
3rd mode	1.47	3.00

The results reported in Table 3 confirm that the proposed algorithm does to a large extent translate into very acceptable, yet very good modal parameter estimation performance even over signals with very closely spaced frequency modes which has not been done before.

7. Conclusions

Unlike the classical random decrement technique, for which the filtering process is an essential preliminary step for estimating the random decrement signature when applying on multi-component signals, this paper demonstrates how to use the Random Decrement Technique without filtering for modal parameter estimations of buildings subjected to ambient excitations. This is done by proposing a new method for modal parameter estimations, which allows dealing with multi-mode Random Decrement Signatures instead. The resulting multi-mode RDS are then used to identify the modal parameters of the signal under study by using a non-parametric method from a time-frequency distribution with bias on the frequency estimations, as an initialization step. This step is then followed by a final non-biased parametric method using the maximum likelihood estimator for an accurate, and robust modal parameter estimations.
The proposed algorithm helped in making the RDT totally autonomous and more friendly user, and the estimation of the modal parameters faster, easier, reliable, and totally independent of user interaction.
In the future we aim to propose a modal parameter modelization method which can estimate directly the number of modes without any initialization step.

Acknowledgements

This work has been supported by French Research National Agency (ANR) through RISKNAT program (project URBASIS ANR-09-RISK-009).

References

1. H. A. Cole, ‘On-the-line Analysis of Random Vibrations’, AIAA: ASME Structures, Structural Dynamics and Materials Conference, Palm Springs (1968).
2. J. K. Vandiver, A. B Dunwoody, R. B Campbell, and M. F. Cook, ‘A Mathematical Basis of the Random Decrement Vibration Signature Analysis Technique’, J. of Mechanical Design, Vol. 104, pp. 307-313, (1982).
3. J.C. Asmuussen, ‘Modal Analysis Based on the Random Decrement Technique: Application to Civil Engineering Structures’, PhD Dissertation, U. of Aalborg, D. (1997).
4. S. R. Ibrahim, ‘Random Decrement Technique for Modal Identification of Structures’, Journal of Spaccecraft and Rockets, vol. 14, No. 11, pp. 696-700 (1997).
5. A. Zubaydi, ‘The Filtering Effect of Random Responses of Stiffened Plates on Their Random Decrement Signatures and Natural Frequencies’, Majalah IPTEK, Vol. 16, No. 1, (2005).
6. F. Nasser, Z. Li, N. Martin, and P. Gueguen, ‘Automatic Parameter Setting of Random Decrement Technique for the Estimation of Building Modal Parameters’, Surveillance 7 International conference, Chartres, France, Oct 2013.

7. J. Antoni and M. El Badaoui, 'The Discrete-Time Random Decrement Technique: Closed-form Solutions for the Blind Identification of SIMO Systems', International Conference on SSI (2011).
8. J.C.S Yang, N. Dagalakis, and M. Hirt, 'Application of the Random Decrement Technique in the Detection of an Induces Crack on an Offshore Platform Model, Computer Methods for Offshore Structures'. ASME, 165-21, pp. 55-67, (1980).
9. N. Martin and M. Vieira, 'Frequency and amplitude tracking for short nonstationary and nonlinear signals', 5th Inter. Conf. on Condition Monitoring and Machinery Failure Prevention Technologies, pp. 15–18, Edinburgh, UK, July 2008.
10. J.C. Asmussen, S.R. Ibrahim, and R. Brincker, 'Random Decrement: Identification of Structures Subjected to Ambient Excitation'. In Proceedings of the 16th International Modal Analysis Conference *(IMAC 16),* Santa Barbara, CA, 914-921, 1998.